7.8 Output signal-to-noise ratios for analog systems

Output SNR for analog system

In this section, we will evaluate the output signal-to-noise ratios for analog systems (AM, DSB-SC, SSB, PM, FM)

For systems with additive noise channels the input to the receiver is

$$r(t) = s(t) + n(t)$$

For bandpass communication systems having a transmission bandwidth of B_T,

$$r(t) = \text{Re}\left\{g_s(t)e^{j(\omega_c t+\theta_c)}\right\} + \text{Re}\left\{g_n(t)e^{j(\omega_c t+\theta_c)}\right\}$$

$$= \text{Re}\left\{[g_s(t) + g_n(t)]\, e^{j(\omega_c t+\theta_c)}\right\}$$

or

$$r(t) = \text{Re}\left\{g_T(t)e^{j(\omega_c t+\theta_c)}\right\}$$

Output SNR for analog system

where

$$g_T(t) = g_s(t) + g_n(t)$$
$$= [x_s(t) + x_n(t)] + j[y_s(t) + y_n(t)]$$
$$= x_T(t) + jy_T(t)$$
$$= R_T(t)e^{j\theta_T(t)}$$

To compare the SNRs for various types of bandpass systems (AM, DSB-SC, SSB, PM, FM),

***assume*:**

- **The power of the modulated signals at the inputs of these receivers is set to the same value P_s, the bandwidth of the baseband modulating signal is B.**
- **The PSD of the input noise is $N_0/2$**

Comparison with baseband systems

Common measurement criterion

The received signal power P_s divided by the amount of power in the white noise that is contained in a bandwidth equal to the message bandwidth.

$$\left(\frac{S}{N}\right)_{baseband} = \frac{P_s}{N_0 B}$$

AM systems with product detection

The output of the product detector is:

$$\tilde{m}(t) = R_e\{g_T(t)\} = A_c + A_c m(t) + x_n(t)$$

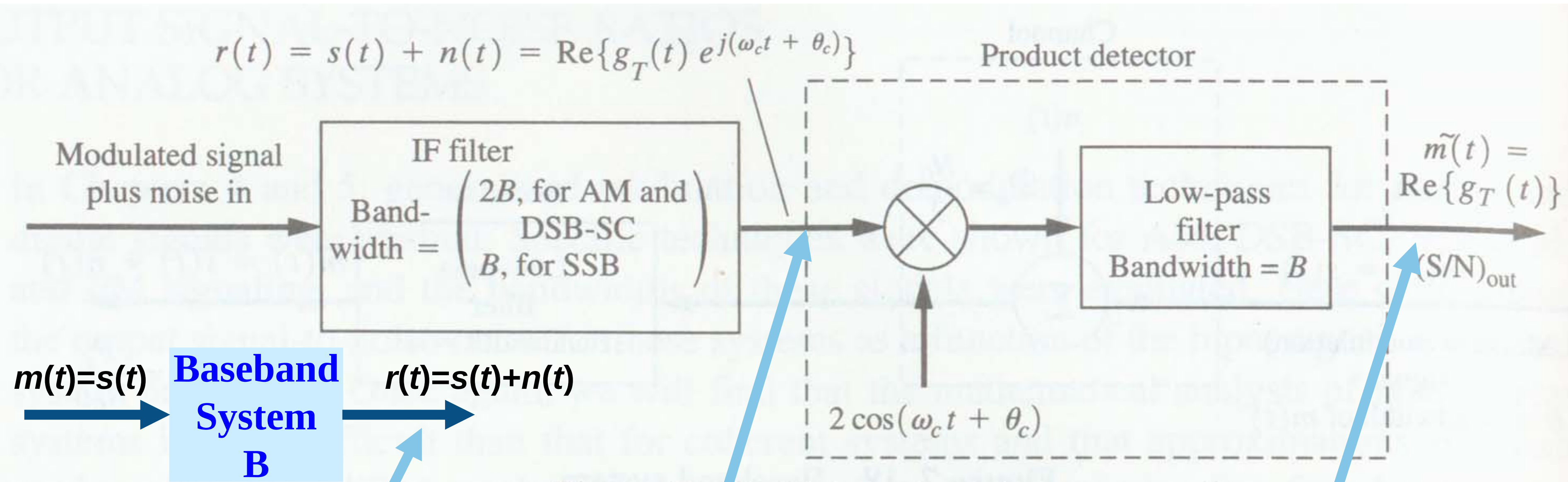


Figure 7–19 Coherent receiver.

The baseband SNR $(S/N)_{baseband}$

The intput SNR $(S/N)_{in}$

The output SNR $(S/N)_{out}$

AM system

The output SNR is:

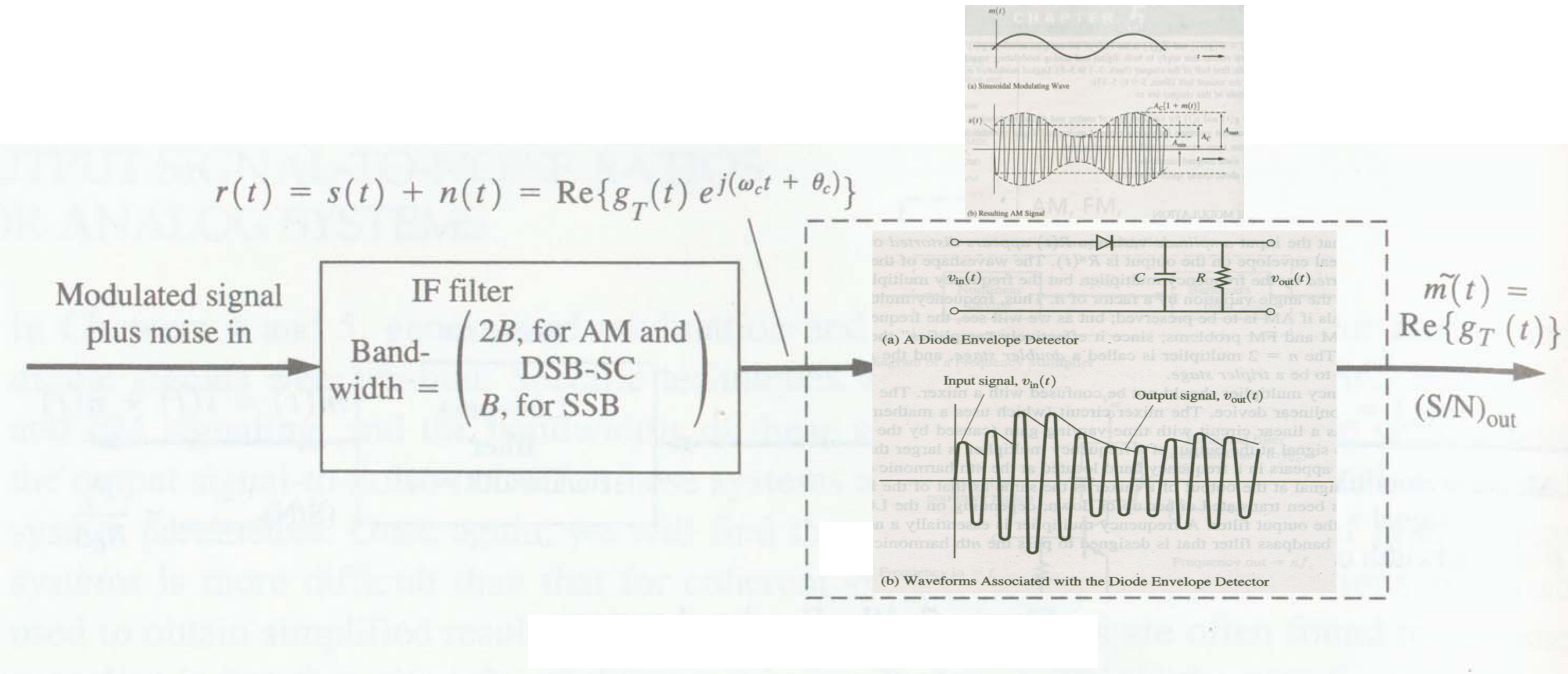
$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2\overline{m^2(t)}}{\overline{x_n^2(t)}} = \frac{A_c^2\overline{m^2(t)}}{2N_0B}$$

$$\frac{(S/N)_{out}}{(S/N)_{in}} = \frac{2\overline{m^2}}{1+\overline{m^2}}$$

$$\frac{(S/N)_{out}}{(S/N)_{baseband}} = \frac{\overline{m^2}}{1+\overline{m^2}}$$

AM system

AM systems with envelope detection



The output of the envelope detector is:

$$\tilde{m}(t) = KR_T(t) = K\left|[A_c + A_c m(t) + x_n(t)] + j[y_n(t)]\right|$$

AM system

For the case of large $(S/N)_{in}$:

The detector output power is:

$$\overline{[KR_T(t)]^2} = (KA_c)^2 + K^2 A_c^2 \overline{m^2} + K^2 \overline{x_n^2}$$

The output SNR is:

$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2 \overline{m^2}}{\overline{x_n^2}} = \frac{A_c^2 \overline{m^2}}{2N_0 B}$$

For large $(S/N)_{in}$, the performance of the envelope detector is identical to that of the product detector

For small $(S/N)_{in}$:

The detector output is:

$$KR_T(t) = K\left|g_T(t)\right| = K\left|A_c[1+m(t)] + R_n(t)e^{j\theta_n(t)}\right|$$

For small $(S/N)_{in}<1$:

$$KR_T(t) \approx K\left\{A_c[1+m(t)]\cos\theta_n(t) + R_n(t)\right\}$$

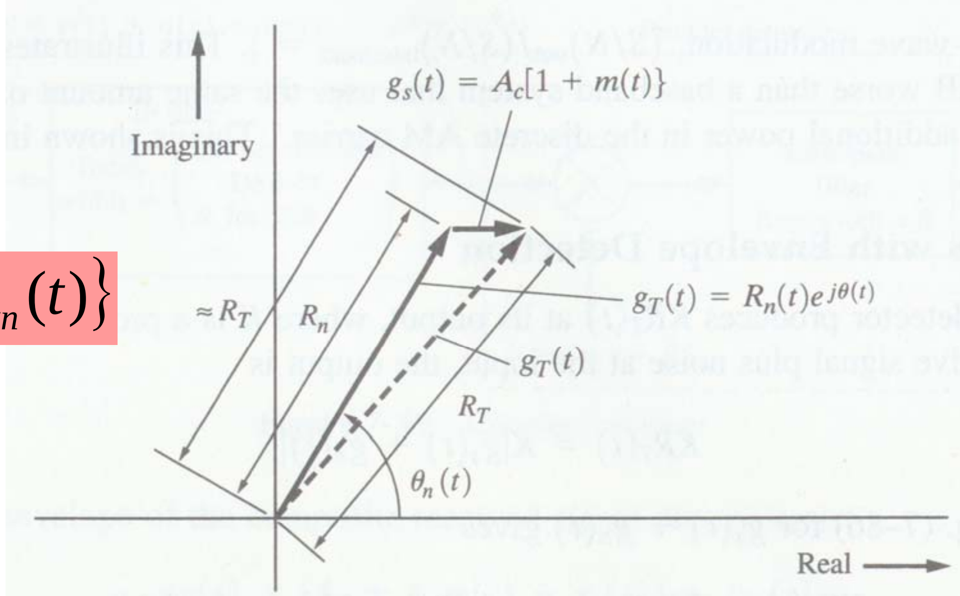


Figure 7–20 Vector diagram for AM, $(S/N)_{in} \ll 1$.

DSB-SC system

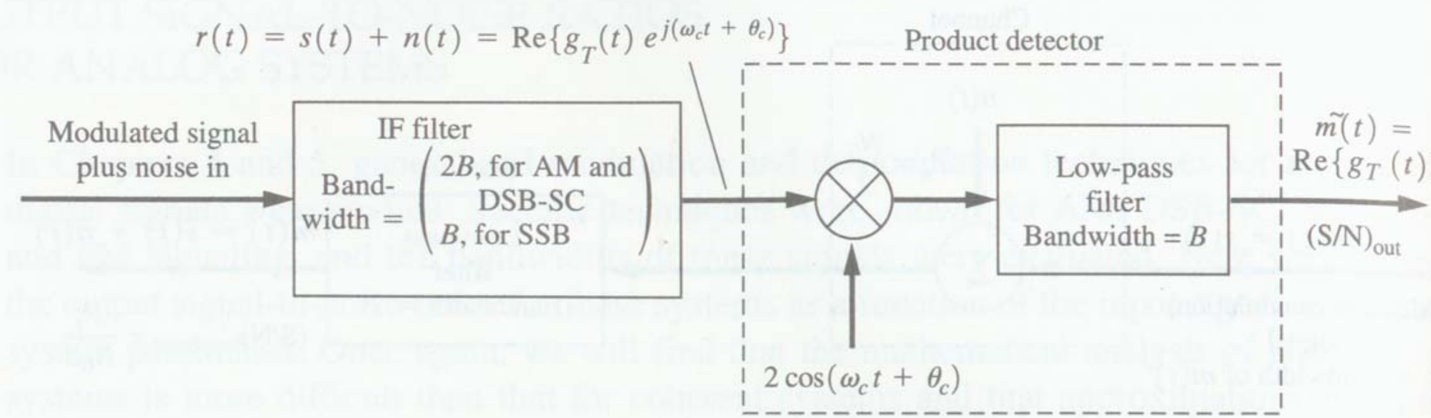


Figure 7–19 Coherent receiver.

DSB-SC signal : $$g_s(t) = A_c m(t)$$

noise : $$g_n(t) = x_n(t) + jy_n(t)$$

signal + noise : $$g_T(t) = [A_c m(t) + x_n(t)] + jy_n(t)$$

The output of the product detector is:

$$\tilde{m}(t) = R_e\{g_T(t)\} = A_c m(t) + x_n(t)$$

DSB-SC system

The SNR for DSB-SC is:

$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2 \overline{m^2(t)}}{2N_0 B}$$

$$\frac{(S/N)_{out}}{(S/N)_{in}} = 2$$

$$\frac{(S/N)_{out}}{(S/N)_{baseband}} = 1$$

The noise performance of a DSB-SC system is the same as that of basedband signaling system, although the bandwidth requirement is twice as large (i.e., B_T=2B)

The receiver for an SSB signal:

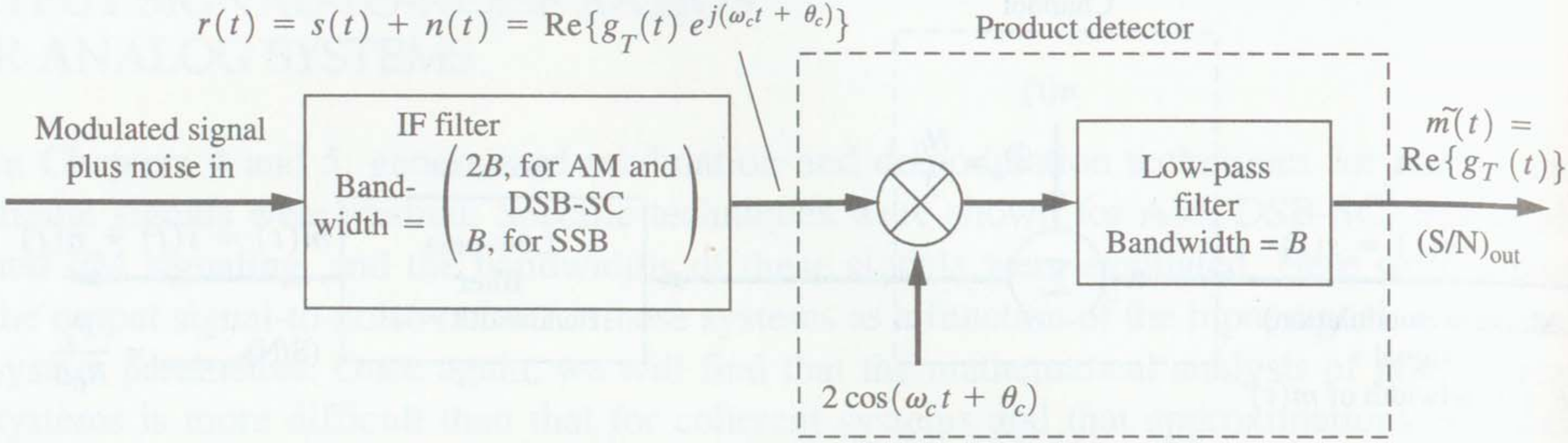


Figure 7–19 Coherent receiver.

The output of the product detector is:

$$\tilde{m}(t) = R_e\{g_T(t)\} = A_c m(t) + x_n(t)$$

SSB system

The output SNR is:

$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2 \overline{m^2(t)}}{\overline{x_n^2(t)}} = \frac{A_c^2 \overline{m^2(t)}}{N_0 B}$$

$$\frac{(S/N)_{out}}{(S/N)_{in}} = 1 \qquad \frac{(S/N)_{out}}{(S/N)_{baseband}} = 1$$

- **SSB is exactly equivalent to baseband signaling, in terms of both the noise performance and the bandwidth requirement (i.e., $B_T=B$)**

- **DSB, SSB and baseband signaling systems are all equivalent in output SNR.**

PM system

The receiver for an PM signal:

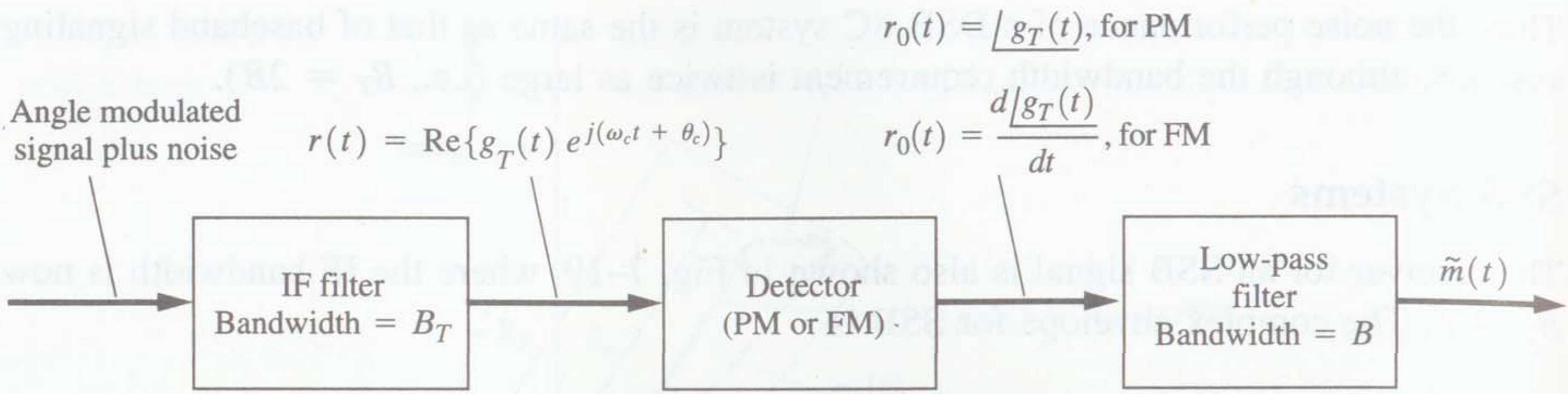


Figure 7–21 Receiver for angle-modulated signals.

The complex envelope at the detector input is:

$$g_T(t) = |g_T(t)| e^{j\theta_T(t)} = [g_s(t) + g_n(t)]$$

$$= A_c e^{j\theta_s(t)} + R_n(t) e^{j\theta_n(t)}$$

PM system

The phase detector output is proportional to $\theta_T(t)$

$$r_0(t) = K\angle g_T(t) = K\theta_T(t)$$

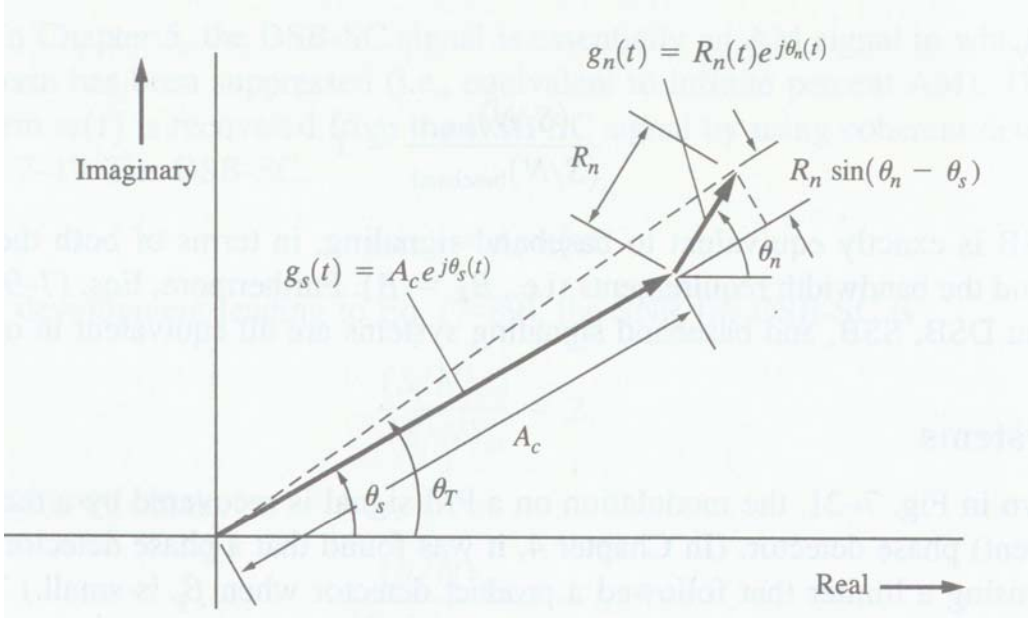


Figure 7–22 Vector diagram for angle modulation, $(S/N)_{in} \gg 1$.

For $A_c \gg R_n(t)$, the composite phase angle is approximated by

$$r_0(t) = K\theta_T(t) \approx K\left\{\theta_s(t) + \frac{R_n(t)}{A_c}\sin[\theta_n(t) - \theta_s(t)]\right\}$$

PM system

For large $(S/N)_{in}$, the relevant part of the PM detector output is approximated by

$$r_0(t) \approx s_0(t) + n_0(t)$$

where

$$s_0(t) = K\theta_s(t) = KD_p m(t)$$

$$n_0(t) = \frac{K}{A_c} y_n(t)$$

The PSD of the output noise $n_0(t)$ is:

$$p_{n_0}(f) = \begin{cases} \frac{K^2}{A_c^2} N_0 \ , & |f| \le B_T/2 \\ 0 \ , & f \ \ otherwise \end{cases}$$

PM system

$$\overline{\tilde{n}_0^2(t)} = \int_{-B}^{B} p_{n_0}(f)\,df$$

$$= \frac{2K^2 N_0 B}{A_c^2}$$

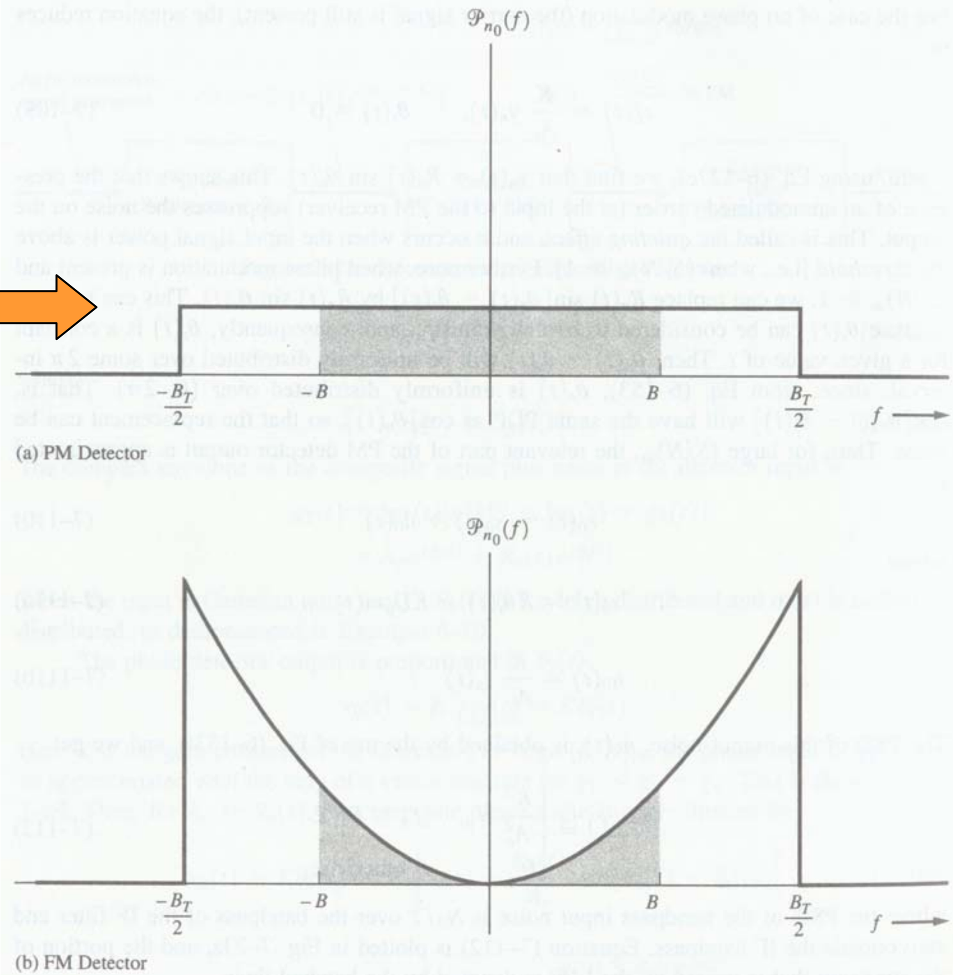


Figure 7–23 PSD for noise out of detectors for receivers of angle-modulated signals.

Using Eqs. (5–46) and (5–47), we can write the sensitivity constant of the PM transmitter

PM system

The output SNR is:

$$\left(\frac{S}{N}\right)_{out} = \frac{\overline{s_0^2}}{\overline{\tilde{n}_0^2}} = \frac{A_c^2 D_p^2 \overline{m^2}}{2N_0 B}$$

Because $D_p = \beta_p / V_p$

Thus, the output SNR becomes:

$$\left(\frac{S}{N}\right)_{out} = \frac{A_c^2 \beta_p^2 \overline{(m/V_p)^2}}{2N_0 B}$$

PM system

The input SNR

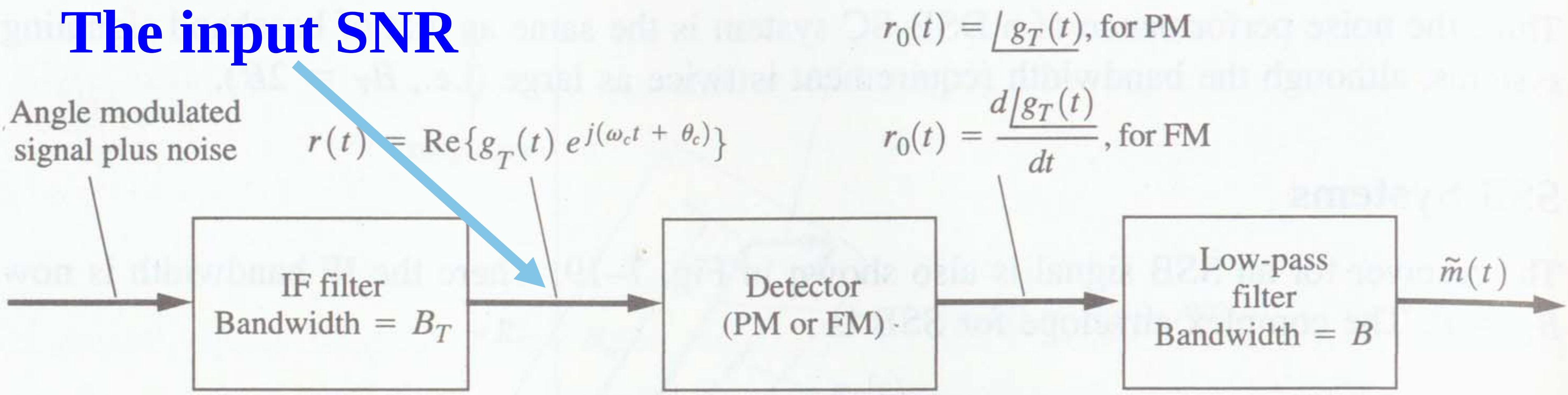


Figure 7–21 Receiver for angle-modulated signals.

$$\left(\frac{S}{N}\right)_{in} = \frac{A_c^2/2}{N_0 B_T} = \frac{A_c^2}{2N_0 B_T}$$

According to Carson's rule:

$$B_T = 2(\beta_p + 1)B$$

$$\left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_0(\beta_p + 1)B}$$

PM system

The ratio of output to input SNR:

$$\frac{(S/N)_{out}}{(S/N)_{in}} = 2\beta_p^2(\beta_p+1)\overline{\left(\frac{m}{V_p}\right)^2}$$

The ratio of output to baseband SNR:

$$\frac{(S/N)_{out}}{(S/N)_{baseband}} = \beta_p^2\overline{\left(\frac{m}{V_p}\right)^2}$$

The improvement of a PM system over a baseband signaling system depends on $\beta_p = \Delta\theta$

The receiver for an FM signal:

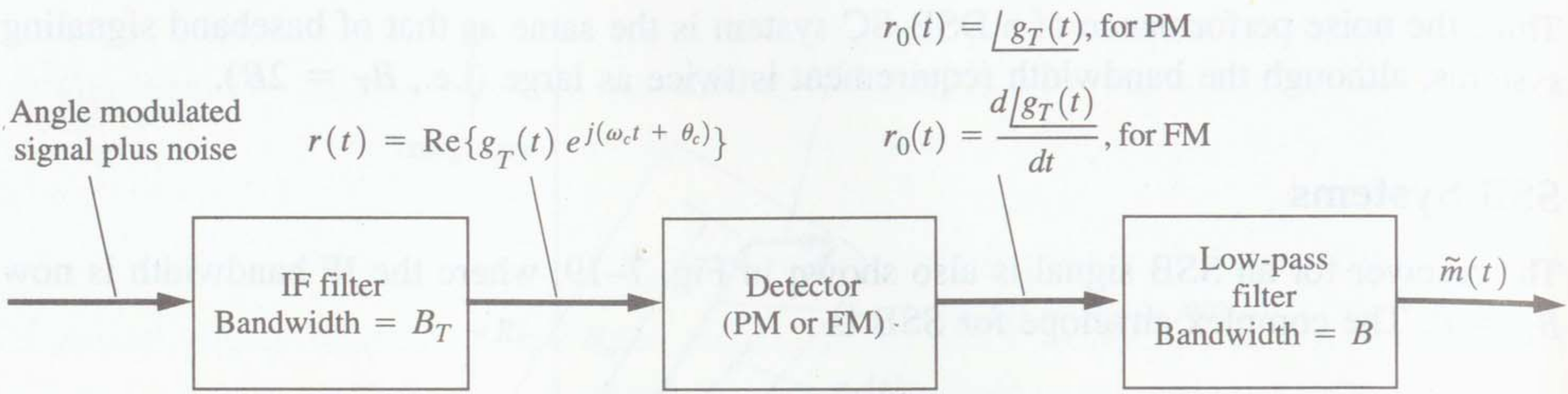


Figure 7–21 Receiver for angle-modulated signals.

The complex envelope at the detector input is:

$$g_T(t) = |g_T(t)| e^{j\theta_T(t)} = [g_s(t) + g_n(t)]$$

$$= A_c e^{j\theta_s(t)} + R_n(t) e^{j\theta_n(t)}$$

FM system

The FM detector output is proportional to the derivative of $\theta_T(t)$

$$r_0(t) = \left(\frac{K}{2\pi}\right)\frac{d\angle g_T(t)}{dt} = \left(\frac{K}{2\pi}\right)\frac{d\theta_T(t)}{dt}$$

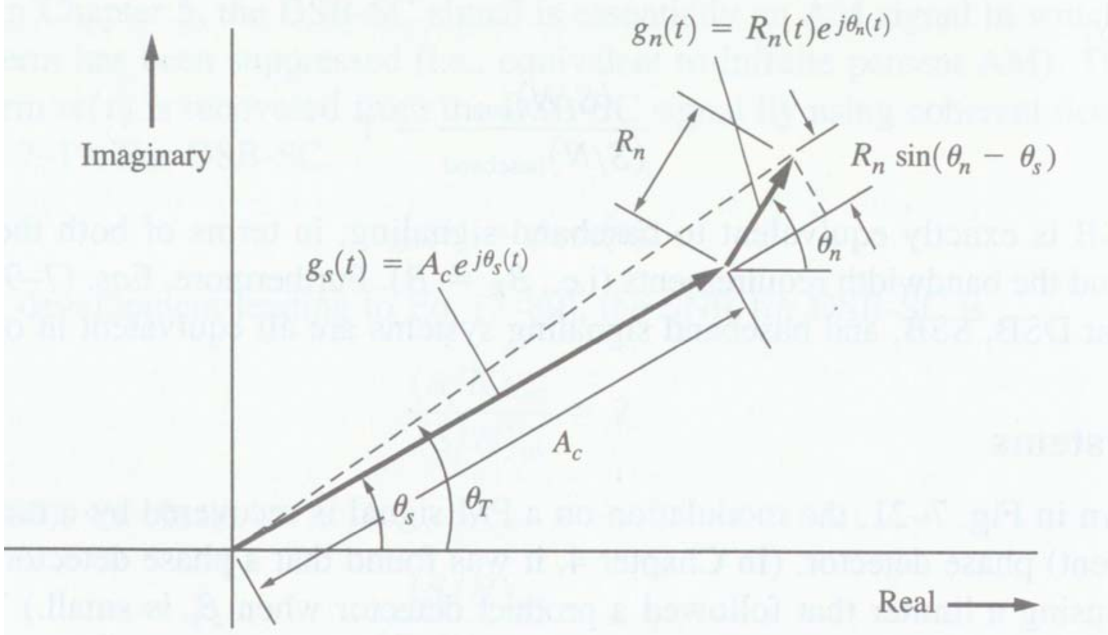


Figure 7–22 Vector diagram for angle modulation, $(S/N)_{in} \gg 1$.

FM system

For large $(S/N)_{in}$>>1, we can approximate the FM detector output by

$$r_0(t) \approx s_0(t) + n_0(t)$$

where

$$s_0(t) = \left(\frac{K}{2\pi}\right)\frac{d\theta_s(t)}{dt} = \left(\frac{KD_f}{2\pi}\right)m(t)$$

$$n_0(t) = \left(\frac{K}{2\pi A_c}\right)\frac{dy_n(t)}{dt}$$

For FM, the PSD of the output noise $n_0(t)$ is:

$$p_{n_0}(f) = \begin{cases} \dfrac{K^2}{A_c^2} N_0 f^2 \;, & |f| < B_T/2 \\ 0 \;, & f \;\; otherwise \end{cases}$$

FM system

$$\overline{\tilde{n}_0^2(t)} = \int_{-B}^{B} p_{n_0}(f)\,df$$

$$= \frac{2}{3}\left(\frac{K}{A_c}\right)^2 N_0 B^3$$

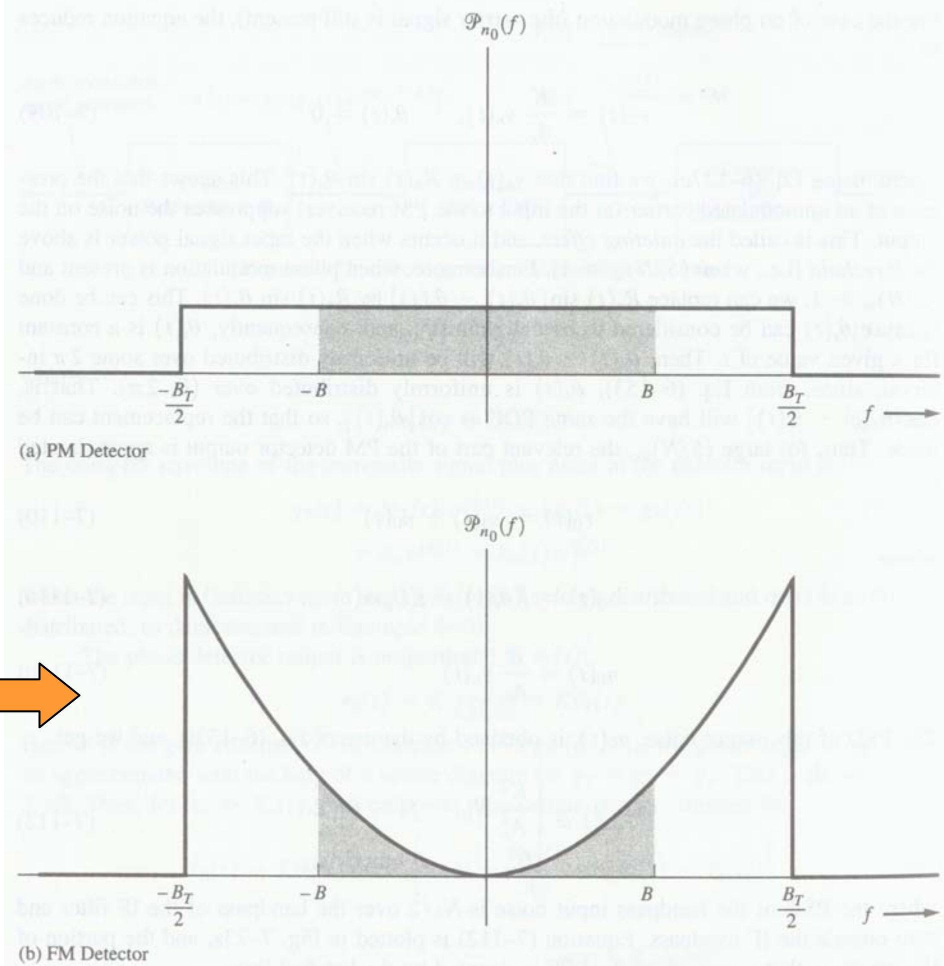


Figure 7–23 PSD for noise out of detectors for receivers of angle-modulated signals.

Using Eqs. (5–46) and (5–47), we can write the sensitivity constant of the PM transmitter

FM system

The output SNR is:

$$\left(\frac{S}{N}\right)_{out} = \frac{\overline{s_0^2}}{\overline{\tilde{n}_0^2}} = \frac{3A_c^2[D_f/(2\pi B)]^2\overline{m^2}}{2N_0B}$$

Because $$\frac{D_f}{2\pi B} = \frac{\beta_f}{V_p}$$

Thus, the output SNR becomes:

$$\left(\frac{S}{N}\right)_{out} = \frac{3A_c^2\beta_f^2\overline{(m/V_p)^2}}{2N_0B}$$

FM system

The input SNR is:

$$\left(\frac{S}{N}\right)_{in} = \frac{A_c^2/2}{(N_0/2)(2B_T)} = \frac{A_c^2}{2N_0 B_T}$$

The transmission bandwidth B_T of the FM signal is given by Carson's rule:

$$B_T = 2(\beta_f + 1)B$$

Thus:

$$\left(\frac{S}{N}\right)_{in} = \frac{A_c^2}{4N_0(\beta_f + 1)B}$$

FM system

The ratio of output to input SNR:

$$\frac{(S/N)_{out}}{(S/N)_{in}} = 6\beta_f^2(\beta_f + 1)\overline{\left(\frac{m}{V_p}\right)^2}$$

The ratio of output to baseband SNR:

$$\frac{(S/N)_{out}}{(S/N)_{baseband}} = 3\beta_f^2\overline{\left(\frac{m}{V_p}\right)^2}$$

For the case of sinusoidal modulation, $\overline{(m/V_p)^2} = 1/2$**,**

$$\frac{(S/N)_{out}}{(S/N)_{baseband}} = \frac{3}{2}\beta_f^2$$

FM system

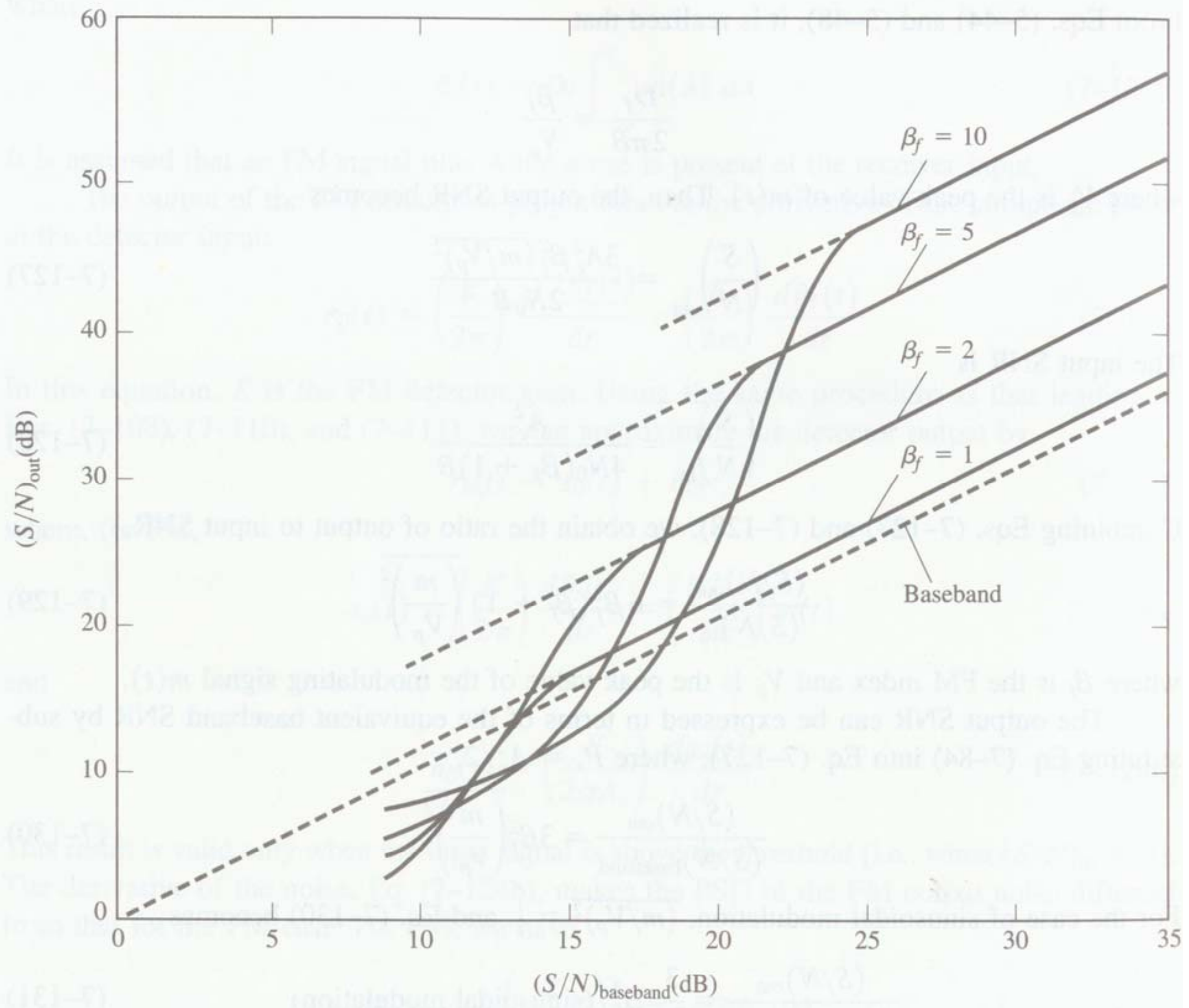


Figure 7–24 Noise performance of an FM discriminator for a sinusoidal modulated FM signal plus Gaussian noise (no deemphasis).

7.9 comparison of analog signaling systems

Comparison

Type	Linearity	Transmission Bandwidth Required[b]	$\frac{(S/N)_{out}}{(S/N)_{baseband}}$		Comments
Baseband	L	B	1	(7–84)	No modulation
AM	L[c]	$2B$	$\frac{\overline{m^2}}{1+\overline{m^2}}$	(7–90)	Valid for all $(S/N)_{in}$ with coherent detection; valid above the threshold for envelope detection and $\lvert m(t)\rvert \le 1$
DSB-SC	L	$2B$	1	(7–98)	Coherent detection required
SSB	L	B	1	(7–105)	Coherent detection required; performance identical to baseband system
PM	NL	$2(\beta_p+1)B$	$\beta_p^2\overline{\left(\frac{m}{V_p}\right)^2}$	(7–120)	Coherent detection required; valid for $(S/N)_{in}$ above the threshold
FM	NL	$2(\beta_f+1)B$	$3\beta_f^2\overline{\left(\frac{m}{V_p}\right)^2}$	(7–130)	Valid for $(S/N)_{in}$ above the threshold
FM with deemphasis	NL	$2(\beta_f+1)B$	$\beta_f^2\left(\frac{B}{f_1}\right)^2\overline{\left(\frac{m}{V_p}\right)^2}$	(7–140)	Valid for $(S/N)_{in}$ above the threshold
PCM	NL	d	$M^2/(S/N)_{baseband}$	(7–82)	Valid for $(S/N)_{in}$ above the threshold (i.e., $P_e \to 0$)

Comparison

- **For the case of large $(S/N)_{in}$:**
  - **The nonlinear modulation systems provide significant improvement in the noise performance.**
  - **But the improvement is obtained at the expense of having to use a wider transmission bandwidth.**
- **For small $(S/N)_{in}$:**
  - **The linear systems outperform the nonlinear systems.**
  - **SSB is the best in terms of small bandwidth, and it has one of the best noise characteristics at low input SNR.**

Comparison

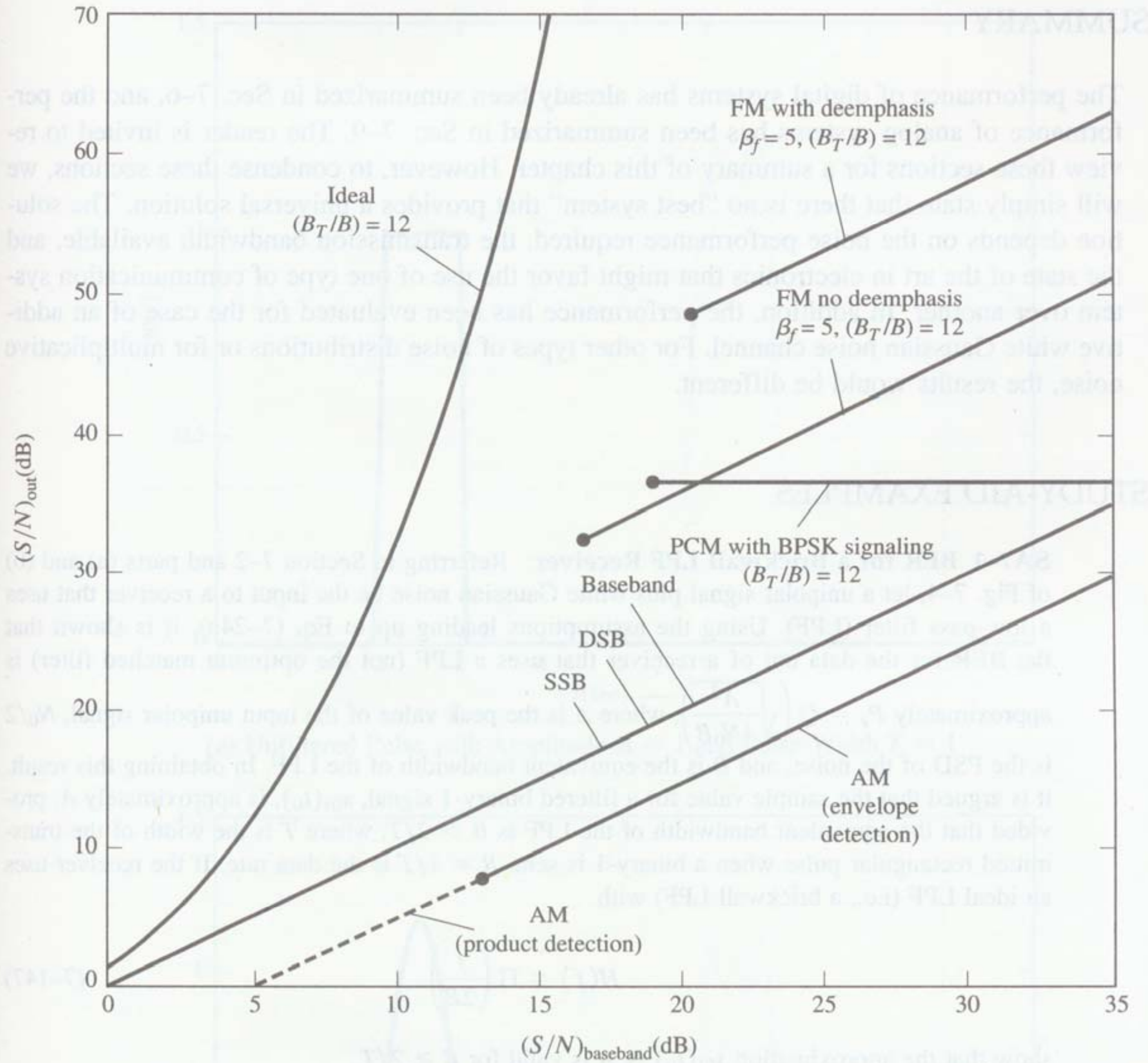


Figure 7–27 Comparison of the noise performance of analog systems.